CHAPTER 2 PROBLEMS AND EXERCISES

Table 1 T300-914 CFRP unidirectional composite stiffness matrix and density

$$\mathbf{C} = \mathbf{C}' = \begin{bmatrix} 143.8 & 6.2 & 6.2 & 0 & 0 & 0 \\ 6.2 & 13.3 & 6.5 & 0 & 0 & 0 \\ 6.2 & 6.5 & 13.3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3.4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5.7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5.7 \end{bmatrix} \text{GPa}, \qquad \rho = 1560\ \text{kg/m}^3$$

Table 2 Typical fiber and matrix properties (www.matweb.com)

	T300 carbon fiber	S-glass fiber	914 epoxy resin
Tensile modulus, GPa	231	86.9	3.90
Poisson ratio	0.20	0.20	0.41
specific density	1.76	2.49	1.29

Table 3 Typical CFRP engineering elastic properties

	E_L, GPa	E_T, GPa	G_{LT}, GPa	G_{23}, GPa	ν_{LT}
T300-914 CFRP	140.0	10.05	5.70	3.40	0.313

PROBLEM 1: FROM STIFFNESS TENSOR TO STIFFNESS MATRIX

Given the stiffness tensor $\mathbf{c}$, do the following:

(a) verify the symmetry properties of the stiffness tensor $\mathbf{c}$

(b) find the stiffness matrix $\mathbf{C}$

Numerical example: T300-914 CFRP unidirectional composite material with $\mathbf{c}$ given as

c(:,:,1,1) = 1.0e+11 * 1.4380 0 0 0 0.0620 0 0 0 0.0620	c(:,:,1,2) = 1.0e+09 * 0 5.7000 0 5.7000 0 0 0 0 0	c(:,:,1,3) = 1.0e+09 * 0 0 5.7000 0 0 0 5.7000 0 0
c(:,:,2,1) = 1.0e+09 * 0 5.7000 0 5.7000 0 0 0 0 0	c(:,:,2,2) = 1.0e+10 * 0.6200 0 0 0 1.3300 0 0 0 0.6500	c(:,:,2,3) = 1.0e+09 * 0 0 0 0 0 3.4000 0 3.4000 0
c(:,:,3,1) = 1.0e+09 * 0 0 5.7000 0 0 0 5.7000 0 0	c(:,:,3,2) = 1.0e+09 * 0 0 0 0 0 3.4000 0 3.4000 0	c(:,:,3,3) = 1.0e+10 * 0.6200 0 0 0 0.6500 0 0 0 1.3300

The units of $\mathbf{c}$ are Pa.

PROBLEM 2: FROM STIFFNESS MATRIX TO STIFFNESS TENSOR

Given the stiffness matrix $\mathbf{C}$, find the stiffness tensor $\mathbf{c}$.

Numerical example: T300-914 CFRP unidirectional composite material, Table 1.

PROBLEM 3: ISOTROPIC COMPLIANCE AND STIFFNESS MATRICES

Given the engineering material properties of an isotropic material such as aluminum and steel, do the following:

(a) find the compliance matrix $\mathbf{S}$

(b) find the stiffness matrix $\mathbf{C}$

(c) verify that the compliance matrix and stiffness matrix are in inverse relationship, i.e., $\mathbf{C} = \mathbf{S}^{-1}$

Numerical values:

	Aluminum (7075-T6)	Steel (AISI 4340 normalized)
Elastic modulus, E	71.7 GPa	205 GPa
Poisson ratio, ν	0.33	0.29

http://asm.matweb.com/search/SpecificMaterial.asp?bassnum=MA7075T6
http://asm.matweb.com/search/SpecificMaterial.asp?bassnum=M434AE

PROBLEM 4:
ENGINEERING PROPERTIES EXTRACTED FROM STIFFNESS MATRIX

Consider a unidirectional composite material given by its stiffness matrix $\mathbf{C}$. Do the following:

(a) recall the transversely isotropic relations that must exist between certain elements of the stiffness matrix $\mathbf{C}$ and verify that they are satisfied numerically.

(b) calculate the compliance matrix $\mathbf{S}$

(c) recall the transversely isotropic relations that must exist between certain elements of the compliance matrix $\mathbf{S}$ and verify that they are satisfied numerically

(d) extract the engineering constants $E_L, E_T, G_{LT}, \nu_{LT}, \nu_{23}$

(e) calculate G_{23} from the appropriate element of the stiffness matrix $\mathbf{C}$ and verify that it is compatible with the engineering constants deduced at item (d) above.

Numerical example: T300/914 CFRP, Table 1

PROBLEM 5:
CFRP PROPERTIES ESTIMATED FROM FIBER AND MATRIX PROPERTIES

Recall the formulae for estimating the elastic properties $E_L, E_T, \nu_{LT}, G_{LT}, G_{23}$ of a composite using the fiber and matrix properties and the volume fraction v. In addition, develop a formula for estimating the composite density ρ.

Numerical example: calculate these properties for a 60% fiber volume fraction T300-914 CFRP composite with the fiber and matrix properties given in Table 2.

PROBLEM 6:
GFRP PROPERTIES ESTIMATED FROM FIBER AND MATRIX PROPERTIES

Recall the formulae for estimating the elastic properties $E_L, E_T, \nu_{LT}, G_{LT}, G_{23}$ of a composite using the fiber and matrix properties and the volume fraction v. In addition, develop a formula for estimating the composite density ρ.

Numerical example: calculate these properties for a 60% fiber volume fraction S-glass-914-epoxy GFRP composite with the fiber and matrix properties given in Table 2.

PROBLEM 7:
VOLUME FRACTION ESTIMATION FROM COMPOSITE PROPERTIES

Estimate volume fraction v_f from the composite properties knowing the fiber and matrix properties:

(a) estimate v_f from density

(b) calculate the compliance matrix $\mathbf{S}$ and the engineering elastic properties

(c) estimate v_f from elastic properties as follows

1. from E_L, E_T
2. from G_{LT}, G_{LT}^{CAM}
3. from G_{23}
4. from ν_{LT}

(d) discuss your results

Numerical example: T300-914 CFRP, Table 1.

PROBLEM 8: PLOT ESTIMATED ENGINEERING PROPERTIES

Plot the estimated engineering elastic properties $E_L, E_T, \nu_{LT}, G_{LT}, G_{23}$ and density ρ vs. volume fiber fraction v_f for CFRP and GFRP composites. Discuss your results.

Numerical example: fiber and matrix properties from Table 2; fiber volume fraction up to 80%.

PROBLEM 9: APPROXIMATE PROPERTY ESTIMATORS

For CFRP and GFRP composites, plot vs. volume fiber fraction v_f the approximation formulae given in Section 2.3.4.6 for estimating the engineering elastic properties in comparison with the formulae given in preceding sections. Discuss your results.

Numerical example: fiber and matrix properties from Table 2; fiber volume fraction up to 80%.

PROBLEM 10: ESTIMATE 3D AND 2D COMPLIANCE AND STIFFNESS MATRICES

(a) Estimate the 3D compliance and stiffness matrices from the engineering properties.
(b) Estimate the 2D compliance and stiffness matrices from the engineering properties
(c) Discuss your results

Numerical example: engineering properties of T300-914 CFRP unidirectional composite of Table 3.

PROBLEM 11: ROTATE 2D COMPLIANCE AND STIFFNESS MATRICES

(a) Calculate the rotated 2D compliance matrix for a range of θ values.
(b) Calculate the rotated 2D stiffness matrix for a range of θ values.
(c) Calculate the rotated 3D stiffness matrix directly using the $\mathbf{T}$ matrix and compare the results with that the result is same with that of item (b)
(d) Discuss your results

Numerical example: T300-914 CFRP unidirectional composite and $\theta = 0°, 30°, 45°, 60°, 90°$

PROBLEM 12: PLOT ELEMENTS OF THE ROTATED 2D STIFFNESS MATRIX

Plot the variation with θ of *Q*(1,1), Q(2,2), *Q*(3,3), *Q*(1,3), *Q*(2,3). Discuss your results

Numerical example: T300-914 CFRP unidirectional composite: $\theta = 0°, ..., 90°$

PROBLEM 13: ROTATE 3D COMPLIANCE AND STIFFNESS MATRICES

(a) Calculate the rotated 3D compliance matrix for a range of θ values.
(b) Use the rotated 3D compliance matrix to calculate the rotated 3D stiffness matrix
(c) Calculate the rotated 3D stiffness matrix directly using the $\mathbf{T}$ matrix and compare the results with that the result is same with that of item (b)
(d) Discuss your results

Numerical example: T300-914 CFRP unidirectional composite and $\theta = 0°, 30°, 45°, 60°, 90°$

PROBLEM 14: PLOT ELEMENTS OF THE ROTATED 3D STIFFNESS MATRIX

Plot the variation with θ of *C*(1,1), *C*(2,2), *C*(4,4), *C*(5,5), *C*(6,6), *C*(1,6), *C*(2,6). Discuss your results

Numerical example: T300-914 CFRP unidirectional composite; $\theta = 0°, ..., 90°$